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(54) Title: ADAPTIVE FILTER FOR A DIRECT SEQUENCE SPREAD SPECTRUM RECEIVER

(57) Abstract

A direct sequence code division multiple access (DS–CDMA) receiver comprises an adaptive filter (8) controlled by an adaptive algorithm (10) for filtering data which has been multiplied at (2) by a spreading code, the filter having a length equal to the number of chips in the code, and a multiuser detector (14) operating on the output of the adaptive filter. Preferably, either the fast a–posteriori error sequential technique (FAEST) algorithm or the stabilised FAEST (SFAEST) algorithm is used as the algorithm (10).

ADAPTIVE FILTER FOR A DIRECT SEQUENCE SPREAD SPECTRUM RECEIVER

Background to the Invention

The present invention relates to a telecommunications receiver employing a new direct sequence code division multiple access (DS-CDMA) architecture which allows the use of fast adaptive algorithms.

Two adaptive algorithms are commonly in use, the LMS and RLS Algorithms[1,2,3] and these are described in Appendix I.

The least mean square (LMS) algorithm (and the closely related normalised least mean squares (NLMS) algorithm) is a stochastic gradient algorithm which has only one parameter, the step size μ. The LMS algorithm is computationally simple but its convergence rate is slow and highly dependent on the properties of the input signal, more specifically on the eigenvalue ratio of the autocorrelation matrix. When many elements of the input signal are unknown, for example the channel in a mobile communications system, it is difficult to choose μ. The algorithm is numerically stable, but an inappropriate choice of μ can cause instability. In high noise conditions, the eigenvalue ratio of the autocorrelation matrix is low and this can help with convergence.

The recursive least squares (RLS) algorithm is computationally much more complex, but has much faster convergence than the LMS algorithm. It has two parameters, the forgetting factor λ and the initial diagonal matrix term δ. The forgetting factor is set appropriate to the rate of change of the autocorrelation of the input signal. The diagonal term has little effect on the algorithm once converged, but does affect the size of internal variables within the algorithm during initial convergence. The RLS algorithm is usually considered converged within a number of iterations equal to twice the filter length, which is generally much faster than the LMS algorithm. The RLS

algorithm can become numerically unstable when the
autocorrelation matrix of the input signal is close to being
a singular matrix.

There are a few much less common adaptive filter
5 algorithms and the use of these algorithms has been found
desirable. The fast a-posteriori error sequential technique
(FAEST)[5,6] algorithm and its stabilised version the SFAEST[7],
which are also described in Appendix I, have a convergence
rate close to the RLS algorithm but complexity close to the
10 LMS algorithm. They do however impose an additional
constraint: the input signal must have a shift invariant
property. The shift invariant property simply means that
the input signal must be the same as the input signal on the
previous iteration shifted on by one sample, with only one
15 new sample. This property is not satisfied by the
conventional architecture for a minimum mean square error
(MMSE) receiver for a DS-CDMA system[8]. The numerical
stability of the FAEST algorithms is not as well understood
as for LMS and RLS, but in practice the SFAEST algorithm
20 seems to remain stable for a sufficiently long period of
time for the purpose proposed here. The Fast Newton
algorithm (see Appendix I) is an algorithm which can
simplify the calculation of any of the above adaptive filter
algorithms if the input signal can be modelled as an
25 autoregressive filter with order less than is assumed by the
above filters.

The conventional architecture for the uplink and
downlink of a DS-CDMA system with an adaptive filter
receiver is shown in Figure 1. In this architecture, the
30 training of an adaptive FIR filter 1 of length N+P-1 chips
(N being the number of chips per data bit and P the total
number of chips in the code) is done at the bit rate, using
an adaption error found by the algorithm to be the
difference between data from a particular user and a sampled
35 estimate of the data from the output of the filter 1, i.e.
the filter has an effective training path ETP. The contents

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of the filter 1 change completely from one iteration thereof
to the next. This means that convergence is slow and it is
not possible to use the FAEST or SFAEST algorithm because
the shift invariance property is not satisfied. With the
5 LMS algorithm, convergence is too slow and the time taken to
reconverge when a user switches on or off is far too slow.
This architecture does work reasonably well with the RLS
algorithm, although convergence is still not very rapid and
the computational complexity is very high.

10 **Summary of the Invention**

 It is one aim of the present invention to provide a
DS-CDMA receiver using an adaptive filter in which the
convergence is rapid. It is another aim of the invention to
allow the use of the less common adaptive algorithms which
15 has not hitherto been possible.

 The present invention provides a direct sequence code
division multiple access (DS-CDMA) receiver comprising an
adaptive filter controlled by an adaptive algorithm for
filtering data which has been multiplied by a spreading code
20 and filtered by a channel filter, the adaptive filter having
a length appropriate to model the inverse of the channel
filter, and a multiuser detector operating on the output of
the adaptive filter.

 The algorithm is preferably either trained using the
25 spread-multiplied signal of a desired user only, or from a
composite signal which is the sum of the spread-multiplied
signals of more than one, for example all, transmitting
users. This means that the adaptive filter will be trained
by new information at the chip rate of the code.

30 In a particular embodiment of the invention the fixed
multiuser detector is of the minimum mean squared error
(MMSE) type, but it may alternatively be of the zero forcing
(decorrelating), Volterra, Radial Basis function,
cancellation, near optimum or other decoding types.

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The algorithm can for example comprise the least mean squares (LMS) or recursive least squares (RLS) algorithm. However, because the adaptive filter of the invention satisfies the shift invariance property, the algorithm may
5 alternatively comprise the fast a-posterion error sequential technique (FAEST) algorithm, the stabilised FAEST (SFAEST) algorithm, and the above algorithms or others may be used in combination with the Fast Newton algorithm.

Brief Description of the Drawings

10   In order that the present invention may be more readily understood, reference will now be made, by way of example only, to the accompanying drawings, in which:-

Figure 1 shows the conventional DS-CDMA architecture already discussed;

15   Figure 2 shows DS-CDMA architecture according to an embodiment of the invention;

Figures 3, 4 and 5 are graphs of simulation results, showing respectively the comparative convergence of the architectures of Figures 1 and 2, the relative convergence
20 rates of different algorithms using the architecture of Figure 2, and the bit error ratio (BER) results for the architecture of Figure 2;

Figure 6 schematically shows a DS-CDMA system with no channel model;

25   Figures 7 and 8 are graphs showing signal to noise performance of a Wiener filter calculated for 7 chip and 31 chip Gold codes respectively;

Figures 9 and 10 are graphs showing bit error rate (BER) perfomance of the Wiener filter calculated for 7 chip
30 and 31 chip Gold codes respectively;

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Figure 11 is a graph showing convergence properties of the LMS and RLS adaptive filters;

Figure 12 is a graph showing BER performance of the LMS and RMS algoritms after 1000 iterations allowed for
5 convergence, compared with the Wiener optimal and a matched filter with no MAI;

Figures 13 and 14 are graphs plotting BER against number of active users in an AWGN channel and a stationary multipath channel respectively, all users being equal power
10 and the spreading code length being 7;

Figure 15 schematically shows the construction of a received signal $Y(n)$;

Figures 16 a) to d) schematically show the structures of a matched filter, a parallel canceller using matched
15 filters, a Wiener filter and a parallel canceller using Wiener filters respectively;

Figure 17 is a graph showing the BER performance of the filters shown in Figures 16 a) to d); and

Figures 18 a) to d) are graphs plotting simulated BER
20 averaged over all users against number of users for four differnt signal to additive Gaussian noise ratios, with 60,000 data bits per user and a sequence length of 64.

Deatiled Description of the Preferred Embodiments
25   Figure 2 shows DS-CDMA transmitter and receiver architecture comprising spreading means 2 respectively operated by each user in which a data signal from the user is multiplied by one of a set of spreading codes uniquely allocated to the respective user. The data is supplied to
30 the spreading means at its bit rate and the code is input at its chip rate, there being N code chips for each data bit.

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The spread-multiplied data is summed at 4 and filtered in a finite impulse response (FIR) channel filter 6 of length P chips, the output of which is transmitted, with incidental Gaussian noise, to a receiver comprising an
5   adaptive FIR filter 8, also of length P chips.

An adaptive algorithm 10 controls and trains the adaptive filter 8. The algorithm 10 basically finds an error signal which is the difference between (a) either the spread-multiplied data from the desired user only or the
10 composite spread-multiplied signal, and (b) the output of the adaptive filter 8. The choice of training data in (a) can either be preset or switchable. The effective training paths ETP for the adaptive filter, representing this operation of the algorithm are shown in Figure 2.

15   The adaptive filter 8 acts similarly to (but not exactly the same as) a conventional equaliser. As previously stated, training can be on the desired user's signal only, or on the composite chip rate signal. The filter 8 has less taps than the adaptive filter 1 in the
20 conventional architecture and is trained at the chip rate, which means much faster convergence. The adaptive filter 8 also satisfies the shift invariance property required for the FAEST and SFAEST algorithm and this allows LMS complexity levels with RLS convergence rates. A fixed
25 multiuser detector 14 is precalculated and stored at the receiver and is based on knowledge obtained about the number of users in the system and their spreading codes. This detector 14 can be of the MMSE type[9] (used in the comparisons here) or it can be of a different type, for example zero
30 forcing or decorrelating[10,] Voterra[11,] Radial Basis function[11]. cancellation based[13, 14] or near optimum decoding based on Viterbi decoding[15].

Summary of Multiuser Detection Techniques

The conventional single user detector or spreading

code matched filter calculates the correlation between the received signal and the spreading code (or the spreading code convolved with the channel impulse response in a multipath channel) over the data bit period. The multiple

5 access interference (MAI) impacts the ability to recover the desired transmissions with this receiver and is more pronounced for high power interfering users.

The matched filter is the optimum detector in AWGN but the MAI cross-correlation terms are not Gaussian unless

10 there are many active users. In this situation the optimal detector is not the conventional matched filter but rather a form of multi-user detector.

The detector that yields the most likely transmitted sequence maximizes the probability that it was transmitted.

15 When all possible transmitted sequences are equally probable, this is the maximum likelihood sequence estimator (MLSE). When implemented with a Viterbi algorithm the complexity is exponential in the number of users. The MLSE must also estimate the received amplitudes and phases but it

20 lowers the mobile user transmitter power control accuracy requirement. Despite the performance and capacity gains over conventional detection, the MLSE is not a practical solution.

Here the zero forcing or decorrelating detector

25 implements an inverse of the received signal autocorrelation matrix (neglecting the noise) such that the output is completely free of multiple access interference. The decorrelating detector was initially proposed in the late 1970s and it provides substantial performance/capacity gains

30 over the conventional detector and does not need to know in advance the received signal amplitudes, avoiding sensitivity to estimation error. It has computational complexity significantly lower than MLSE. Another desirable feature is that it corresponds to the MLSE when the user energies are

35 unknown. A disadvantage of this detector is that it causes

noise enhancement i.e. the power associated with the noise term at the output of the decorrelating detector is always greater or equal to the noise term at the output of the conventional detector. A more significant disadvantage of the decorrelating detector is that the computations needed to invert the matrix are considerable.

A possibly superior linear receiver structure is to build an adaptive filter that minimises (at its output) the error power. This implements a partial or modified inverse of the autocorrelation matrix, dependent on the level of background noise, to balance the desire to decouple the users for MAI reduction while not enhancing the background noise. Again this receiver structure implements a matrix inversion operation and here we can apply the recursive adaptive filter techniques. This detector differs from the decorrelating detector in that it takes the noise terms into account to minimise the noise enhancement. Thus if noise is low the minimum mean squared error (MMSE) receiver approaches the decorrelating detector and achieves close to optimum performance. On the other hand, if the multiple access interference is small compared to the noise, then the matched filter detector solution is approached. This is exactly analogous to the MMSE linear equalizer used to combat inter-symbol interference. Unlike the decorrelating detector, it requires estimation of the received amplitudes but its complexity is independent of number of active users and explicit knowledge of the CDMA spreading sequences is not required for an adaptive implementation.

Other researchers have investigated radial basis function (which in its complete implementation is similar to MLSE, various simplifications have been suggested) and Volterra nonlinear approaches (where the received signal is passed through a power series nonlinear expansion before applying a linear filter eg. decorrelating or MMSE). Note that if a MMSE error type fixed multiuser detector is used, the overall impulse response of the adaptive filter followed

by the fixed detector is not exactly the same as for the conventional architecture even when both systems are fully converged. This will give different bit error ratio (BER) results even when both the adaptive filters in the two

5 receivers are fully converged. Note also that the adaptive filter 8 has an extremely low signal to noise ratio at its output as the processing gain has not been applied at this stage. This leads to a low eigenvalue spread which can help the convergence and stability of some adaptive algorithms

10 but others exhibit instability in the extremely high noise conditions

Possible multiuser detectors are discussed in more detail in Appendix II.

The output from the detector 14 is down-sampled to the

15 bit rate at the synchronous points to give the required estimate of the user's data signal.

Simulation Results

All simulation results presented below are for 16 chip spreading codes (i.e. P=16) and a six tap stationary

20 channel. Firstly we show that the new architecture has far faster or convergence than the conventional one. Figure 3 shows convergence curves for the conventional architecture of Figure 1 and the new architecture of Figure 2. The new architecture is much faster than the conventional one,

25 mostly because it is trained at the chip rate instead of the bit rate but also because the eigenvalue ratio of the autocorrelation is reduced in high noise.

The graph of Figure 3 shows ensemble averaged squared error at the output of the adaptive filter in both

30 architectures plotted against time measured in chips, for the single user case. Both curves use the LMS algorithm with the value of μ individually optimised for each. The use of chip rate training instead of bit rate training,

increases the convergence by a factor of 16, but in fact the convergence is more than 16 times faster because high noise reduces the eigenvalue ratio in the autocorrelation. The new algorithm converges to a MMSE 16 times higher than the

5 conventional architecture, but this loss will be largely regained through the fixed multiuser detector.

Figure 4 shows the relative convergence rates of some different adaptive filter algorithms using the architecture of Figure 2. The use of the SFAEST algorithm is only

10 possible in the new architecture, as the conventional one does not satisfy the shift invariance property.

Relatively speaking, the LMS algorithm is very slow. It can be made faster by increasing the value of μ but when this value is increased too far, the LMS algorithm does not

15 converge to the MMSE error floor (misadjustment error). The RLS is fast, but at the price of high computational complexity. The SFAEST algorithm is as quick as the RLS algorithm if it is initialised appropriately.

Figure 5 shows BER results for the new architecture

20 after allowing 160 chips (only 10 databits) for convergence of the adaptive filter.

With only 160 databits for convergence, the LMS algorithm is not fully converged when the training period ends and this results in a slightly poorer performance than

25 is obtainable with the conventional architecture. The RLS and SFAEST give similar results.

The architecture of the receiver of the invention is much faster converging and tracking than the conventional architecture with only a small loss in BER performance for

30 a stationary channel. In a time varying channel, average BER will be better for the new architecture because of reduced tracking errors. The fast adaptive algorithms (FAEST, SFAEST, preferably in combination with Fast Newton)

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allow the new architecture to achieve good convergence without the computational complexity of the RLS algorithm.

The receiver of the invention could be integrated in a "hard-wired" form or could be made capable of being
5 updated by using reconfigurable or replaceable firmware.

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APPENDIX I
Summaries of Adaptive Algorithms

Least mean square algorithm

From the stochastic gradient approach [1] one obtains
5 the well known least mean squares (LMS) algorithm which was
first introduced in 1960 by Widrow and Hoff [2]. This
algorithm is uncomplicated and yields acceptable performance
in most cases. The LMS technique is probably the most
frequently used adaptive algorithm in current communication
10 system. It is derived by applying the method of steepest
descent minimisation to the Wiener-Hopf equations which
define the optimum Wiener filter and one obtains a simple
recursive scheme of the form [3]

$$\left\{ \begin{array}{c} \text{new} \\ \text{tap weights} \end{array} \right\} = \left\{ \begin{array}{c} \text{old} \\ \text{tap weights} \end{array} \right\} + \left\{ \begin{array}{c} \text{learning} \\ \text{rate} \end{array} \right\} \left\{ \begin{array}{c} \text{new} \\ \text{information} \end{array} \right\} \tag{1}$$

where the new information consists of the product of the
15 filter input vector and the error signal, i.e. the
difference between the desired filter output and the actual
output of the filter. Essentially one can express the LMS
algorithm (and many other adaptive algorithms) by means of
three expressions [1]:

20   Filter output: $y(t) = \mathbf{w}(t)^T \mathbf{x}(t)$ (2)

Adaption error: $e(t) = d(t) - y(t)$ (3)

Tap weight update: $\mathbf{w}(t+1) = w(t) + \mu \mathbf{x}(t)e(t)$ (4)

where $\mathbf{x}$ is the filter input vector, $\mathbf{w}^T$ the transposed tap
weight vector, d represents the desired filter output and μ
25 is the learning rate (step size parameter).

Several modifications have been made to improve the
LMS algorithm, the most important one resulted in the
normalised LMS (NLMS [4]) which normalises the adaption
error e using instantaneous estimates of the input vector
30 power $\|\mathbf{x}\|^2$. Doing so largely improves the algorithm's
stability characteristics and allows faster convergence. The
LMS algorithm is computationally simple but its convergence

rate is slow and highly dependent on the properties of the
input signal, more specifically on the eigenvalue ratio of
the autocorrelation matrix. When many elements of the input
signal are unknown, for example the channel in a mobile
5 communications system, it is difficult to choose μ. The
algorithm is numerically stable, but an inappropriate choice
of μ can cause instability. In high noise conditions, the
eigenvalue ratio of the autocorrelation matrix is low and
this can help with convergence. Other versions of the LMS
10 such as leaky, signed and quantised LMS [3] were developed
but only the NLMS achieved sufficient stability at
acceptable performance and is therefore considered as a
candidate for application in advanced multiuser detection.

        To illustrate the improved performance of the
15 normalised LMS as compared to the standard LMS we will
present the performance of both the LMS and NLMS later in
this document report.

Recursive least square algorithm
        The most popular least-square (LS) technique is is
20 probably the recursive least squares (RLS) algorithm. The
RLS algorithm is computationally much more complex, but has
much faster convergence than the LMS algorithm. It has two
parameters, the forgetting factor λ and the initialisation
factor δ for the diagonal matrix $P(0)$, see Equations below.
25 The forgetting factor is set appropriate to the rate of
change of the autocorrelation of the input signal. The
diagonal term has little effect on the algorithm once
converged, but does effect the size of internal variables
within the algorithm during initial convergence. The RLS
30 algorithm is usually considered converged within a number of
iterations equal to twice the filter length, which is
generally much faster than the LMS algorithm. The RLS
algorithm can become numerically unstable when the
autocorrelation matrix of the input signal is close to being
35 a singular matrix.

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We can summarise the standard RLS algorithm as follows
[1]:

INITIALISATION

$$P(0) = \delta^{-1} I$$
$$w(0) = 0$$

SUBSEQUENT ITERATIONS

$$g(t) = \frac{P(t-1)x(t)}{\lambda + x^T(t)P(t-1)x(t)} \tag{5}$$

$$e(t) = d(t) - w_N^T(t-1)x_N(t) \tag{6}$$

$$w(t) = w(t-1) + g(t)e(t) \tag{7}$$

$$P(t) = \frac{1}{\lambda}\left(P(t-1) - g(t)x^T(t)P(t-1)\right) \tag{8}$$

5   The vector $g(t)$ is known as the gain vector of the
algorithm, $P(t)$ represents the inverse of the correlation
matrix

$$\Phi(t) = \sum_{i=1}^{t} \lambda^{t-i} x(t) x(t)^T$$

which can be computed in a recursive manner by
means of the matrix inversion lemma. The variable e denotes
10 the a-priori error of the filter estimation and the filter
weights are again represented by the vector w.

Fast a-posteriori error sequential technique (FAEST)

The fast a-posteriori error sequential technique
(FAEST) has first been reported by Carayannis et. al [5] in
15 the context of least-square (LS) filtering. Compared to
standard LS algorithms the FAEST uses a different approach
for calculating the Kalman gain vector based on the
a-posteriori error formulation rather than the a-priori
error formulation as used in fast Kalman algorithms [6].
20 Assuming the shift invariance property of the input signal
and introducing a slightly modified version of the Kalman
gain, the FAEST algorithm manages to perform a direct
updating of the Kalman gain without invoking matrix-vector
multiplications.

25   The Kalman gain update according to the FAEST
algorithm is summarised in the Table 1.

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Table 1: The FAEST algorithm.

Time update of gain vector	Multiplications	Divisions
$e_N^f(t) = x(t) - a_N^*(t-1) x_N(t-1)$	N	–
$\varepsilon_N^f(t) = \frac{e_N^f(t)}{\gamma_N(t)}$	–	1
$\alpha_N^f(t) = \lambda \alpha_N^f(t-1) + e_N^f(t) \varepsilon_N^f(t)$	2	–
$\gamma_{N+1}(t) = \gamma_N(t) \frac{\alpha_N^f(t)}{\lambda \alpha_N^f(t-1)}$	2	1
$w_{N+1}(t) = \begin{bmatrix} 0 \\ w_N(t-1) \end{bmatrix} - \frac{e_N^f(t)}{\gamma \alpha_N(t-1)} \begin{bmatrix} 1 \\ -a_N(t-1) \end{bmatrix}$	$N+1$	1
$a_N(t) = a_N(t-1) - \varepsilon_N^f(t) w_N(t-1)$	N	–
$e_N^b(t) = -\lambda \alpha_N^b(t-1) w_{N+1}^{N+1}(t)$	2	–
$\theta(t) = 1 + e_N^b(t) \frac{w_{N+1}^{N+1}(t)}{\gamma_{N+1}(t)}$	1	1
$\gamma_N(t) = \theta(t) \gamma_{N+1}(t)$	1	–
$\varepsilon_N^b(t) = \frac{e_N^b(t)}{\gamma_N(t)}$	–	1
$\alpha_N^b(t) = \lambda \alpha_N^b(t-1) + e_N^b(t) \varepsilon_N^b(t)$	2	–
$\begin{bmatrix} w_N(t) \\ 0 \end{bmatrix} = w_{N+1}(t) - w_{N+1}^{N+1} \begin{bmatrix} -b_N(t-1) \\ 1 \end{bmatrix}$	N	–
$b_N(t) = b_N(t-1) - \varepsilon_M^b(t) w_N(t)$	N	–
Total number of multiplications and divisions	$5N + 11$	5

It is well known that the FAEST algorithm suffers from sever stability problems and we will therefore not describe this algorithm in more detail but focus on its stabilised
5 version, the SFAEST. For more information about FAEST please refer to [5].

Stabilised FAEST

The stabilised version of the FAEST algorithm has been derived by [7] and is presented in Table 3. For easier
10 reading of the equations building the stabilised FAEST (SFAEST) algorithm, we will first list all variables occurring and explain their meaning with a few words, see Table 2.

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Table 2: Variable definitions of the SFAEST algorithm.

Variable Name	Definition
$x(t)$	Filter input at time t
$y(t)$	Filter output, $y(t) = 0 \, \forall t < 0$
$\mathbf{x}_N(t)$	Input vector $[x(t), \ldots, x(t - N + 1)]$
$\mathbf{g}_N(t)$	Dual Kalman gain vector
$\mathbf{a}_N(t)$	Forward predictor coefficients
$\mathbf{b}_N(t)$	Backward predictor coefficients
$\mathbf{h}_N(t)$	Filter coefficients
$e_N(t)$	A-priori filter error
$\epsilon_N(t)$	A-posteriori filter error
$e_N^f(t), \bar{e}_N^f(t)$	Forward predictor error and its corrected version
$e_N^b(t), \bar{e}_N^b(t)$	Backward predictor error and its corrected version
$\alpha_N^f(t)$	Forward prediction error power
$\alpha_N^b(t)$	Backward prediction error power
$\gamma_N(t), \gamma_{N+1}(t)$	$0 \leq \gamma_N(t) \leq 1$
$\theta_N(t)$	
$\xi_N(t), \bar{\xi}_N(t)$	Difference of errors and the corrected error difference
λ	exp. forgetting factor $0 < \lambda \leq 1$

Table 3 shows the successive steps of the SFAEST algorithm and also evaluates the complexity in terms of multiplications and divisions required.

Table 3: Tasks of SFAEST in order of computations and number of required MUL/DIV operations

Tasks of SFAEST	MUL	DIV
Available at time t: $g_N(t-1)$, $a_N(t-1)$, $b_N(t-1)$, $h_N(t-1)$, $x_N(t-1)$ $\gamma_N(t-1)$, $\alpha_N^f(t-1)$, $\alpha_N^b(t-1)$		
New Information: $x(t)$, $y(t)$		
Computation of the residuals and corrections: A-priori forward/backward errors (residuals): 1) $\quad e_N^f(t) = x(t) - a_N(t-1)^T x_N(t-1)$ 2) $\quad e_N^b(t) = x(t-N) - b_N(t-1)^T x_N(t)$	 N N	 - -
Normalisation parameters: 3) $\quad \gamma_{N+1}(t) = \frac{\lambda \alpha_N^f(t-1)}{\lambda \alpha_N^f(t-1) + \gamma_N(t-1)(e_N^f(t))^2} \gamma_N(t-1)$	 4	 1
4) $\quad \theta_N(t) = 1 + \gamma_{N+1}(t)e_N^b(t)\left(g_N^N(t-1) + \frac{e_N^f(t)}{\lambda \alpha_N^f(t-1)} a_N(t-1)^N\right)$	3	1
5) $\quad \gamma_N(t) = \frac{\gamma_{N+1}(t)}{\theta_N(t)}$ and $\gamma_N(t) = \frac{1}{1-g_N^f(t)x_N(t)}$	-	1
6) $\quad k_N(t-1) = \lambda^{-N}\gamma_N(t-1)a_N^N(t-1)$	2	-
Difference of errors: 7) $\quad \xi_N(t) = e_N^b(t) + k_N(t-1)e_N^f(t) + \lambda \alpha_N^b(t-1)g_N^N(t-1)$	 3	 -
Difference of errors of corrected filters 8) $\quad \bar{\xi}_N(t) = \frac{\xi_N(t)}{1+\rho(1-\gamma_N(t))+\rho k_N^2(t-1)(1-\gamma_N(t-1))}$	 4	 1
Corrected a-priori forward/backward errors: 9) $\quad \bar{e}_N^f(t) = e_N^f(t) - (1-\gamma_N(t-1))k_N(t-1)\rho\bar{\xi}_N(t)$ 10) $\quad \alpha_N^f(t) = \lambda \alpha_N^f(t-1) + \gamma_N(t-1)(\bar{e}_N^f(t))^2$ 11) $\quad \bar{e}_N^b(t) = e_N^b(t) - (1-\gamma_N(t))\rho\bar{\xi}_N(t)$ 12) $\quad \alpha_N^b(t) = \lambda \alpha_N^b(t-1) + \gamma_N(t)(\bar{e}_N^b(t))^2$	 3 3 2 3	 - - - -
Time update of the *dual* Kalman gain $g_N(t)$: Extended Kalman gain vector: 13) $\quad g_{N+1}(t) = \begin{bmatrix} 0 \\ g_N(t-1) \end{bmatrix} - \frac{\bar{e}_N^f(t)}{\lambda \alpha_N^f(t-1)} \begin{bmatrix} 1 \\ -a_N(t-1) \end{bmatrix}$	 $N+1$	 1
Forward filter: 14) $\quad a_N(t) = a_N(t-1) - \gamma_N(t-1)(e_N^f(t) + k_N(t-1)\rho\bar{\xi}_N(t))g_N(t-1)$	 $N+4$	 -
Dual Kalman gain: 15) $\quad \begin{bmatrix} g_N(t) \\ 0 \end{bmatrix} = g_{N+1}(t) - g_{N+1}^{N+1}(t) \begin{bmatrix} -b_N(t-1) \\ 1 \end{bmatrix}$	 N	 -
Backward filter: 16) $\quad b_N(t) = b_N(t-1) - \lambda_N(t)(e_N^b(t) + \rho\bar{\xi}_N(t))g_N(t)$	 $N+3$	 -
Time update of the filter $h_N(t)$: 18) $\quad e_N(t) = y(t) - h_N^T(t)x_N(t)$ 19) $\quad \epsilon_N(t) = \gamma_N(t)e_N(t)$ 20) $\quad h_N(t) = h_N(t-1) - \epsilon_N(t)g_N(t)$	 N 1 N	 - - -
Total number of multiplications and divisions:	$8N+36$	

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Initialisation, stabilisation and optimisation issues related to the SFAEST

The value of the forgetting factor λ largely depends on the rate by which the input signal changes and defines
5 the memory length of the algorithm. The eigenvalue spread of the weighted input covalence matrix given by

$$R_N(t) = \sum_{\tau=0}^{t} \lambda^{t-\tau} x_{N,\tau} x_{N,\tau}^T \qquad (9)$$

plays an important part in determining λ. A suitable value could for example be $\lambda = 0.98$ but the type of the input signal needs to be considered and it is most likely that
10 changing between static and dynamic users and static and dynamic channels will require different values for λ.

The choice for the variable ρ is not straight forward and no direct rule for calculating an appropriate value exists. In [7] it is mentioned that a initialisation can be
15 done by means of an estimate such as

$$\rho = \rho_0 \frac{\lambda}{1-\lambda} \qquad \text{with} \qquad \rho_0 \approx 0.05. \qquad (10)$$

This could however not be confirmed by simulations carried out in the course of this work. Usually a value of $\rho = 1$ has been chosen but an adaptive estimation of ρ during operation of the algorithm might prove advantageous. The most crucial
20 parameters were found to be the forward/backward error powers. Initialisation has been done by means of the following rules:

$$\alpha_N^f(0) = \mu \lambda^N \qquad (11)$$

$$\alpha_N^b(0) = \mu \qquad (12)$$

As can be seen both error powers depend on the value μ and hence this parameter needs to be initialised with care. A
25 typical, stable range for μ was found to be between $10 < \mu < 100$. From a stability point of view it is advantageous to monitor the evolution of the variable $\gamma_N(t)$. To prevent divergence of the algorithm, this variable should be restricted to values between 0 and 1. A recommended rule [7]

to assure convergence is to reinitialise the algorithm when
the condition

$$\xi_N^2(t) > v\alpha_N^b(t) \tag{13}$$

becomes true, where v is a small constant. This precaution
will also take care of the case where the gain tends towards
5   zero, hence $\gamma_N \rightarrow 0$.

Fast Newton transversal filter

        The Fast Newton algorithm is an algorithm which can
simplify the calculation of any of the above adaptive filter
algorithms if the input signal can be modelled as an
10 autoregressive filter with order less than is assumed by the
above filters. Fast Newton transversal filters originate
from the area of speech enhancement and echo cancellation.
Their main feature is a fast calculation of the gain vector
as required in many LS adaptive algorithms.

15   The stabilised fast Newton transversal filter (SFNTF)
is essentially a computational accelerator for any least
square (LS) algorithm. Operating as a "higher-level"
adaptive predictor it can use any LS algorithm as a
subroutine within its own algorithm. However, the order of
20 this LS filter can be chosen to be smaller than the actual
filter order of the SFNTF algorithm. A sophisticated
predictor part then extrapolates the remaining filter
coefficients to gain the complete set of filter coefficients
as required according to the definition of the SFNTF length.
25 It is this feature that should make the SFNTF a potentially
attractive adaptive algorithm for many application and the
usefulness of SFNTFs for advanced MUD receivers shall be
examined in the future. As far as this report is concerned,
we will however restrict ourselves to the description of the
30 algorithm only.

        To discuss the details of the SFNTF we will again list
all variables as present in the algorithm with some words of

explanation about the meaning of the variables. Table 4 lists these definitions.

Table 4: Variable definitions of the SFNTF algorithm.

Variable Name	Definition
$x(t)$	Filter input at time t
$y(t)$	Filter output, $y(t) = 0 \, \forall t < 0$
$\mathbf{x}_N(t)$	Input vector $[x(t), ..., x(t - N + 1]$
$\mathbf{g}_N(t)$	Dual Kalman gain vector of order N
$\mathbf{q}_N(t), \mathbf{q}_{N+1}(t)$	temporary vectors to compute $\mathbf{g}_N(t)$ in version 2 and 3.
$\mathbf{s}_{P+1}, \mathbf{u}_{P+1}$	temporary vectors to compute $\mathbf{g}_N(t)$.
$\mathbf{a}_N(t)$	Forward predictor coefficients
$\mathbf{b}_N(t)$	Backward predictor coefficients
$\mathbf{h}_N(t)$	Filter coefficients
$\mathbf{R}_N$	Input sample covariance matrix
$\mathbf{r}_N$	Vector that builds $\mathbf{R}_N$, see end of table.
$e_N(t)$	A-priori filter error
$\epsilon_N(t)$	A-posteriori filter error
$e_N^f(t), \bar{e}_N^f(t)$	Forward predictor error and its corrected version
$e_N^b(t), \bar{e}_N^b(t)$	Backward predictor error and its corrected version
$\alpha_N^f(t)$	Forward prediction error power
$\alpha_N^b(t)$	Backward prediction error powerr
$\gamma_N(t), \gamma_{N+1}(t)$	$0 \leq \gamma_N(t) \leq 1$
$\theta_N(t)$	
$\xi_N(t), \zeta_N(t)$	Difference of errors and the corrected error difference
λ	exp. forgetting factor $0 \leq \lambda \leq 1$

We can now describe the SFNTF with all its equations.
5 For easier reading of the following equations, we first define two new vectors $\mathbf{s}$ and $\mathbf{u}$:

$$\mathbf{s}_{P+1}(t) = \frac{1}{\lambda} \frac{e_P^f(t)}{\alpha_P^f(t-1)} \begin{bmatrix} 1 \\ -\mathbf{a}_P(t-1) \end{bmatrix} \tag{14}$$

$$\mathbf{u}_{P+1}(t) = \frac{1}{\lambda} \frac{e_P^b(t)}{\alpha_P^b(t-1)} \begin{bmatrix} -\mathbf{b}_P(t-1) \\ 1 \end{bmatrix}, \quad t^0 = t - N + P \tag{15}$$

Comparing these definitions with Equation 4, we notice the similarity with the updating part $\mu x e$. While λ represents the forgetting factor similar to μ, the second
10 factor of Equations 14 and 15 are normalised error signals

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and the third factor, containing the vectors **a** and **b**, are similar to the input vector **x** of Equation 4. Note however that this is only a basic attempt to explain the nature of the vectors **s** and **u** and not a completely valid comparison -

5 in particular with respect to the third factor. The vectors **a** and **b** represent the forward and backward predictor coefficients of the FNTF, respectively, and are computed by means of the sample covariance matrix R_P by

$$\mathbf{a}_P(t) = \mathbf{R}_P^{-1}(t-1) \begin{bmatrix} x^1(t) \\ \vdots \\ x^P(t) \end{bmatrix} \tag{16}$$

and

$$\mathbf{b}_P(t) = \mathbf{R}_P^{-1}(t) \begin{bmatrix} x^P(t) \\ \vdots \\ x^1(t-P+1) \end{bmatrix}. \tag{17}$$

10   Their corresponding predictor error powers can then be defined as

$$\alpha_P^f(t) = x^0(t) - \begin{bmatrix} x^1(t) & \ldots & x^P(t) \end{bmatrix} \mathbf{a}_P(t) \tag{18}$$

and

$$\alpha_P^b(t) = x^0(t-P) - \begin{bmatrix} x_1(t) & \ldots & x^P(t-P+1) \end{bmatrix} \mathbf{b}_P(t) \tag{19}$$

As we notice from the definitions above, the SFNTF operates two separate predictor branches and by enabling or

15 disabling those branches we can create three different versions of the algorithm:

Version 1: Using forward and backward predictors (not recommended)

Available values at time t: $g_N(t-1)$ and $\gamma_{N,P}(t-1)$
20 From SFAEST algorithm: $s_{P+1}(t)$, $e_P^f(t)$, $u_{P+1}(t^0)$ and $e_P^b(t^0)$

$$g_{N+1}(t) = \begin{bmatrix} 0 \\ g_N(t-1) \end{bmatrix} - \begin{bmatrix} s_{P+1}(t) \\ 0_{N-P} \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} g_N(t) \\ 0 \end{bmatrix} = g_{N+1}(t) + \begin{bmatrix} 0_{N-P} \\ u_{P+1}(t^0) \end{bmatrix} \tag{21}$$

$$\gamma_{N,P}(t) = \gamma_{N,P}(t-1) + s_{P+1}^1(t)e_P^f(t) - u_{P+1}^{P+1}(t^0)e_P^b(t^0) \tag{22}$$

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Verson 2: Using only forward predictors

Available at time t: $q_N(t-1), g_N(t-1)$
From SFAEST algorithm: $s_{P+1}(t), e_P^f(t), s_{P+1}(t^0), e_P^f(t^0), g_P(t^0), \gamma_P(t^0)$

$$q_{N+1}(t) = \begin{bmatrix} 0 \\ q_N(t-1) \end{bmatrix} + \begin{bmatrix} s_{P+1}(t) \\ 0_{N-P} \end{bmatrix} \tag{23}$$

$$\begin{bmatrix} q_N(t) \\ 0 \end{bmatrix} = q_{N+1}(t) - \begin{bmatrix} 0_{N-P} \\ s_{P+1}(t^0) \end{bmatrix} \tag{24}$$

$$g_N(t) = \begin{bmatrix} 0_{N-P} \\ g_P(t^0) \end{bmatrix} - q_N(t) \tag{25}$$

$$g_N(t) = g_N(t-1) + s_{P+1}^1(t)e_P^f(t) - s_{P+1}^1(t^0)e_P^f(t^0) \tag{26}$$

$$\gamma_{N,P}(t) = \gamma_P(t^0) + g_N(t) \tag{27}$$

Version 3: Using only backward predictors

5 Available at time t: $q_N(t-1), g_N(t-1)$
From SFAEST algorithm: $u_{P+1}(t), e_P^b(t), u_{P+1}(t^0), e_P^b(t^0), g_P(t^0), \gamma_P(t^0)$

$$q_{N+1}(t) = \begin{bmatrix} 0 \\ q_N(t-1) \end{bmatrix} + \begin{bmatrix} u_{P+1}(t) \\ 0_{N-P} \end{bmatrix} \tag{28}$$

$$\begin{bmatrix} q_N(t) \\ 0 \end{bmatrix} = q_{N+1}(t) - \begin{bmatrix} 0_{N-P} \\ u_{P+1}(t^0) \end{bmatrix} \tag{29}$$

$$g_N(t) = \begin{bmatrix} g_P(t) \\ 0_{N-P} \end{bmatrix} - q_N(t) \tag{30}$$

$$g_N(t) = g_N(t-1) + u_{P+1}^{P+1}(t)e_P^b(t) - u_{P+1}^{P+1}(t^0)e_P^b(t^0) \tag{31}$$

$$\gamma_{N,P}(t) = \gamma_P(t) + g_N(t)$$

Time update of the filter $h_N(t)$:

Available at time t: $h_N(t-1), x_N(t-1)$
New information: $x(t), y(t)$

$$e_N(t) = y(t) - h_N^H(t-1)x_N(t-1) \tag{32}$$

$$\epsilon_N(t) = \frac{e_N(t)}{\gamma_{N,P}(t)} \tag{33}$$

$$h_N(t) = h_N(t-1) - \epsilon_N(t)g_N(t) \tag{34}$$

10   Predictor version 1 is more likely to diverge due to the fact that the calculation of γ is not realised as a finite sum. Versions 2 and 3 include only a finite sum of

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P + 1 values in the computation of γ.

As the algorithm of Table 5 is rather complex and difficult to overview, we also include Table 6 which displays the variable occurences in the different equations of Table 5 and states which variables are required for updating other variables.

/ .

[1]Note: The definitions of γ differ between FAEST and FNTF!

Table 5: Tasks of SFNTF with SFAEST in order of computation and number of required MUL/DIV operations

SFNTF with SFAEST	MUL	DIV
Available at time t: $g_P(t-1)$, $a_P(t-1)$, $b_P(t-1)$, $h_{N,t-1}$, $x_P(t-1)$, $\gamma_P(t-1)$, $\alpha_P^f(t-1)$, $\alpha_P^b(t-1)$		
New Information: $x(t)$, $y(t)$		
Computation of the residuals and corrections: A-priori forward/backward errors (residuals) 1) $e_P^f(t) = x(t) - a_P(t-1)^H x_P(t-1)$ 2) $e_P^b(t) = x(t-N) - b_P(t-1)^H x_P(t)$	 P P	 – –
Normalisation parameters 3) $\gamma_{P+1}(t) = \frac{\lambda\alpha_P^f(t-1)}{\lambda\alpha_P^f(t-1)+\gamma_P(t-1)(e_P^f(t))^2}\gamma_P(t-1)$ 4) $\theta_P(t) = 1 \div \gamma_{P+1}(t)e_P^b(t)(g_P^P(t-1) + \frac{e_P^f(t)}{\lambda\alpha_P^f(t-1)}a_P(t-1)^P)$ 5) $\gamma_P(t) = \frac{\gamma_{P+1}(t)}{\theta_P(t)}$ and $\gamma_P(t) = \frac{1}{1-g_P^0(t)x_P(t)}$ 6) $k_P(t-1) = \lambda^{-P}\gamma_P(t-1)a_P^P(t-1)$	 4 3 – 2	 1 1 1 –
Difference of errors: 7) $\xi_P(t) = e_P^b(t) \div k_P(t-1)e_P^f(t) + \lambda\alpha_P^b(t-1)g_P^P(t-1)$	 3	 –
Difference of errors of *corrected* filters 8) $\bar{\xi}_P(t) = \frac{\xi_P(t)}{1 \div \rho(1-\gamma_P(t)) \div \rho k_P^2(t-1)(1-\gamma_P(t-1))}$	 4	 1
Corrected a-priori forward/backward errors 9) $\bar{e}_P^f(t) = e_P^f(t) - (1-\gamma_P(t-1))k_P(t-1)\rho\bar{\xi}_P(t)$ 10) $\alpha_P^f(t) = \lambda\alpha_P^f(t-1) + \gamma_P(t-1)(\bar{e}_P^f(t))^2$ 11) $\bar{e}_P^b(t) = e_P^b(t) - (1-\gamma_P(t))\rho\bar{\xi}_P(t)$ 12) $\alpha_P^b(t) = \lambda\alpha_P^b(t-1) + \gamma_P(t)(\bar{e}_P^b(t))^2$	 3 3 2 3	 – – – –
Time update of the *dual Kalman gain* $g_P(t)$: Extended Kalman gain vector: 13) $g_{P+1,t} = \begin{bmatrix} 0 \\ g_P(t-1) \end{bmatrix} - \frac{\bar{e}_P^f(t)}{\lambda\alpha_P^f(t-1)}\begin{bmatrix} 1 \\ -a_P(t-1) \end{bmatrix}$	 $P \div 1$	 1
Forward filter: 14) $a_P(t) = a_P(t-1) - \gamma_P(t-1)(e_P^f(t) + k_P(t-1)\rho\bar{\xi}_P(t))g_P(t-1)$	 $P+4$	 –
Dual Kalman gain: 15) $\begin{bmatrix} g_P(t) \\ 0 \end{bmatrix} = g_{P+1,t} - g_{P+1,t}^{P+1}\begin{bmatrix} -b_P(t-1) \\ 1 \end{bmatrix}$	 P	 –
Backward filter: 16) $b_P(t) = b_P(t-1) - \gamma_P(t)(e_P^b(t) + \rho\bar{\xi}_P(t))g_P(t)$	 $P \div 3$	 –
Computation of dual Kalman gain for SFNTF: 17) see description of Version 1), 2) and 3)	 2	
Time update of the SFAEST filter $h_P(t)$: (not required for SFNTF) 18a) $e_P(t) = y(t) - h_P^H(t)x_P(t)$ 19a) $\epsilon_P(t) = \gamma_P(t)e_P(t)$ 20a) $h_P(t) = h_P(t-1) - \epsilon_P(t)g_P(t)$	 P 1 P	 – – –
Time update of the SFNTF filter $h_{N,t}$: 18b) $e_N(t) = y(t) - h_{N,t-1}^H x_{N,t-1}$ 19b) $\epsilon_N(t) = e_N(t)/\gamma_N(t)$ [1] 20b) $h_{N,t} = h_{N,t-1} - \epsilon_N(t)g_{N,t}$	 N – N	 – 1 –
Total number of multiplications and divisions:	$8P + 2N + 39$	

Table 6: Variable occurrence in the SFAEST algorithm (Table 5).

Equation in Table 3	Variables as function of $t/(t-1)$																		
	$x(t)$	$x(t-P)$	x_P	w_P	w_{P+1}	i_P	b_P	e_P	e_P^b	α_P^f	α_P^b	γ_P	γ_{P+1}	θ_P	k_P	ξ_P	$\bar{e}_P^f$	$\bar{e}_P^b$	$\bar{\xi}_P$
1)	∘		−/∘			−/∘		•/−											
2)		∘	∘/−				−/∘		•/−										
3)				−/∘		−/∘		∘/−		−/∘		−/∘	•/−	•/−					
4)								∘/−	∘/−	−/∘		•/−	∘/−	∘/−					
5)				−/∘		−/∘						−/∘							
6)				−/∘		−/∘		∘/−		−/∘		−/∘							
7)					•/−			∘/−			−/∘	•/−	−/∘		•/−	•/−	•/−	•/−	•/−
8)					∘/−						∘/∘	−/∘		∘/∘	∘/−	∘/−	∘/−	∘/−	∘/−
9)								∘/−				−/∘		−/∘					
10)									∘/−	•/∘		−/∘						∘/−	∘/−
11)										•/∘		∘/−							
12)								∘/−											
13)				−/∘	•/−	−/∘		∘/−		−/∘					−/∘		∘/−		∘/−
14)				−/∘	∘/−	•/∘													
15)				•/−			−/∘		∘/−			∘/−							∘/−
16)				∘/−			•/∘												
# of variables required	1	1	1	1	1	1	1	1	1	1/2	1	2	1	1	1	1	1	1	1

Legend: • = variable being updated
           ∘ = value of variable required
           − = value not required
           */⋆ = describes the time step $t/t-1$ of variables.
           •/∘ for b_P in Equation 16) denotes $b_P(l)$ will be updated and $b_P(l-1)$ is required.

Example: •/∘ for b_P in Equation 16) denotes $b_P(l)$ will be updated and $b_P(l-1)$ is required.

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APPENDIX II
Multiuser detectors for CDMA

THE MMSE RECEIVER

Consider the direct sequence spread spectrum system shown
5 in Fig. 6.

In it we have M ($m = 1$ to M) users, each transmitting a
PN code of length N chips ($n = 1$ to N). We assume that all the
users are both data bit and chip synchronous and that the data
modulation D_m and the codes C_{mn} take the values ± 1. The signal
10 at the input to the receiver at chip n is therefore:-

$$y(n) = \sum_{m=1}^{M} D_m C_{mn} + W(n) \tag{1}$$

$W(n)$ is the noise sample at chip n. Wiener filter theory
[1], [15] states that the optimal set of weights for an FIR
filter h_{opt} is given by:-

$$\bar{h}_{opt} = \underline{\emptyset}_{yy}^{-1} \bar{\emptyset}_{yx} \tag{2}$$

$\underline{\emptyset}_{yy}$ is the autocorrelation matrix of the input signal and
15 for a stationary input:-

$$\underline{\emptyset}_{yy} = E \begin{bmatrix} y^2(n) & y(n)y(n-1) & .. & y(n)y(n-N) \\ y(n)y(n-1) & y^2(n) & .. & y(n)y(n-N+1) \\ y(n)y(n-2) & y(n)y(n-1) & .. & y(n)y(n-N+2) \\ . & . & & . \\ . & . & & . \\ y(n)y(n-N) & y(n)y(n-N+1) & .. & y^2(n) \end{bmatrix} \tag{3}$$

where $E[x]$ denotes the expected value of x. We are interested
in the FIR filter when it contains no data transitions, ie
when it only has one data bit from each user in it. Assuming
that the data modulation on each code is independent, ie
20 $E(D_a D_b) = 0$, $A \neq B$ by direct substitution of equation {1} into
equation {3} or analogy with the spatial radar case [16], [17]
it can be shown that

$$\underline{\emptyset}_{yy} = \underline{Q} \Lambda \underline{Q}^T + \sigma^2 I \tag{4}$$

The matrix Q has dimension NxM and has the codes as its
columns, ie:-

$$Q = \begin{bmatrix} C_{11} & C_{21} & C_{31} & . & . & C_{M1} \\ C_{12} & C_{22} & C_{32} & . & . & C_{M2} \\ C_{13} & C_{23} & C_{33} & . & . & C_{M3} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ C_{1N} & C_{2N} & C_{3N} & . & . & C_{MN} \end{bmatrix}$$

Q^T denotes the transpose of matrix Q. The matrix Λ has dimensions $M{\times}M$ and has P_m at position m along its leading diagonal and is zero elsewhere. P_m is the power at the receiver of user m. For the rest of the section we shall
5 assume $P_m = 1$ for all m, ie perfect power control with the power of each user normalised to 1. The $\sigma^2 I$ term is the noise term assuming the noise power is σ^2 and the noise is uncorrelated.

The vector $\emptyset_{xy}$ is given by:-

$$\bar{\emptyset}_{xy} = E\left[x(n) \begin{bmatrix} y(n) \\ y(n-1) \\ y(n-2) \\ . \\ . \\ y(n-N) \end{bmatrix} \right] \tag{5}$$

10    $x(n)$ is the desired response. We are only interested in the response when the FIR filter contains the chips from one data bit, ie when $n = 0$ and at that point the desired response is the data bit for the desired code. Assuming the desired code is $m = 1$ then $x(n) = D_1$. Substituting this and equation
15 {1} into equation {5} and assuming the the data symbols are uncorrelated yields:-

$$\bar{\emptyset}_{xy} = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \\ . \\ . \\ C_{1N} \end{bmatrix} \tag{6}$$

Thus the vector $\emptyset_{xy}$ is the desired code. From equations {1}, {3} and {6}, and under the assumptions that we have stated, the optimum mean squared error performance is only
20 dependent on the code set chosen. The minimum mean squared error at the FIR filter output is given by:-

$$\varepsilon_{\min} = E[x^2(n)] - \vec{h}_{opt}^T \bar{\varnothing}_{xy}$$

We can define the output signal to noise ratio for the filter as:-

$$\left(\frac{S}{N}\right)_{out} (dB) = 10 \log_{10}\left(\frac{P_1}{\varepsilon}\right)$$

Assuming that the channel bandwidth in Hz is equal to the chip rate in chips per second then we can use the energy per 5 bit E_b divided by the noise power spectral density N_0 as a processing gain independent measure of the signal to noise ratio at the input to our FIR filter. In our case:-

$$\frac{E_b}{N_0} = \frac{1}{N\sigma^2}$$

Figs. 7, 8, 9 and 10 show the theoretical and simulated performance of the Wiener filter calculated as above for a set 10 of 7 and 31 chip Gold codes. Fig. 7 shows the theoretical and simulated performance of the 7 chip Gold codes in terms of the output signal to noise ratio from the Wiener filter. Fig. 7. shows that the performance of the optimal filter with 4 users is practically the same as the performance of a matched filter 15 with no MAI. Looking at the data, at 10 dB output signal to noise ratio, the difference between the 4 user curve and the matched filter curve is around 0.3 dB. From Fig. 7, to have 7 users in a system with a processing gain of 7 we must have an increase of around 1.8 dB in E_b/N_0 to achieve the same 20 performance as a matched filter with no MAI. These figures are again measured with 10 dB as the required output signal to noise ratio.

Fig. 8 shows theoretical and simulated performance of the 31 chip Gold codes in terms of the output signal to noise 25 ratio from the Wiener filter. With 16 users, we require around a 0.1 dB increase in E_b/N_0 over the matched filter with no MAI. This is better than the 4 user, 7 chip case, partly because 4/7 is greater than 16/31. When there are 31 users in a 31 chip system, the required increase in E_b/N_0 is 1 dB over 30 the matched filter with no MAI. It would appear that the 31

chip system enjoys some inherent advantage over the 7 chip system, probably due to the greater degree of statistical orthogonality between the 31 chip codes when compared with the 7 chip case. However, other factors such as the choice of the

5 desired user or the choice of the code sets cannot be ruled out.

Figs. 9 and 10 show the simulated bit error rate (BER) performance of the 7 and 31 chip codes. These graphs show similar results to Figs. 7 and 8 in terms of the increase in

10 E_b/N_0 required to accommodate more users. A good approximation to Figs. 9 and 10 can be derived from Figs. 7 and 8 assuming that because of the central limit theorem the noise output from the Wiener filters is Gaussian.

Adaptive filter receivers

15   In most cases the DS-CDMA channel is non-stationary and as we have already stated the MAI will vary with the birth and death of signals. Therefore the FIR filter used in our receiver has to be a time varying approximation to the Wiener filter calculated using an adaptive algorithm. We will

20 consider the properties of the two most popular adaptive algorithms, least mean squares (LMS) and recursive least squares (RLS) [1]. We shall use the 7 chip Gold code case as our example, as the 31 chip case would involve considerable extra computation. Fig. 11 shows the convergence properties

25 of the LMS algorithm with two different values of the adaption parameter μ. The graphs show the feedback error ensemble summed over 100 independent trials. With μ = 0.007 the algorithm converges to 20% of its initial value within approximately 20 data bits, but with μ = 0.0007 convergence

30 takes around 100 data bits. In a DS-CDMA cellular system, where the channel varies relatively quickly with respect to the data rate of a single user, this slow convergence will not be acceptable. Fig. 12 shows the BER performance of the LMS algorithm after convergence with the same values of μ as for

35 Fig. 11. It shows that the smaller value of μ produces a good approximation to the Wiener filter, but the performance of the

LMS algorithm with the larger value of μ is around 2 dB worse
in terms of the required signal to noise ratio for a bit error
rate of 10^{-3}. Thus it is difficult to find a value of μ which
will provide a fast enough convergence without introducing a
5 large residual error into the performance of the filter, even
for the 7 chip case. The convergence time will be greater if
the codes are longer. If we consider a DS-CDMA cellular system
with imperfect power control and a multipath channel, then the
eigenvalue spread of the input signal will be increased. This
10 will also have an adverse effect on the convergence time of
the LMS algorithm. More evidence of this is contained in [18].
Thus the LMS algorithm is unsuitable for this application.
Also shown in Fig 11 is the convergence of an RLS algorithm
with $\lambda = 1$ and the diagonal elements of the autocorrelation
15 matrix initially set to 0.001. This shows typical behaviour
for an RLS algorithm, rapid convergence by $2N$. The RLS curve
in Fig. 12 shows that the RLS algorithm does not add any
significant residual error to the Wiener filter solution. Thus
the RLS algorithm is potentially more suited to a DSCDMA
20 cellular system.

Discussion

    In this section we shall look at the assumptions made in
the derivation of the filters and look at applying this type
of receiver to a cellular system.

25   We have assumed that all the channels are data bit and
chip synchronous. If the channels are not chip synchronous,
provided the receiver samples the desired channel
synchronously, any change is likely to be beneficial as the
effective MAI will be reduced. If the channels are not data
30 bit synchronous however, the effect is greatly dethmental.
With transitions of the interfering channels occurring in the
middle of the desired channel data bits, each interfering
channel requires two eigenvectors in the Q matrix. Thus
performance is reduced and the system breaks down when the
35 number of users exceeds $0.5N$ instead of N for the data bit
synchronous case. An interesting case is when the data bits

are almost synchronous, for example as synchronous as the
transmission delays in the system will allow. In this case,
it may be possible to ensure that any transition occurs a
small number of chips (compared with N) from the beginning or
5 end of the desired user's data bit. In this case the
detrimental effect may not be so great, although this
hypothesis remains unproven. We have also assumed perfect
power control. There is some evidence that this type of
adaptive filter structure is tolerant to variations in the
10 power of the received codes [19]. The last assumption we made
is that the data bits are uncorrelated. Provided that the
signals are independently generated, idle channels are
suppressed and that care is taken over the content and timing
of control information, this should be a reasonable
15 assumption.

If we are to apply this receiver structure to a cellular
DS-CDMA system we need to take into account the non-stationary
multipath channel and the birth and death of users. To take
into account the multipath channel, we can use the technique
20 in [20], replacing the impulse response of the spreading
process C_{mn} with B_{mn}, where B_{mn} is the convolution of the
impulse response of the channel with the impulse response of
the spreading process and repeat the analysis. This paper
already describes the theoretical optimal with multipath
25 evaluated for all users of the system simultaneously. The
non-stationary elements of the channel can be taken into
account by blocks of training data with or without decision
feedback in between. The birth and death of signals problem
will be greatly alleviated if all users are constrained to
30 switch on and off only immediately proceeding a training
block. The convergence and tracking properties of the adaptive
algorithm and speed at which the channel varies will determine
the length and frequency of the training blocks. However, by
employing the RLS algorithm (or the covariance form of the
35 least squares algorithm on training blocks [1]), the ratio of
training data to information carrying data can be kept
reasonable.

Some desirable properties of this receiver are that it does not require to know the desired or the interferers spreading codes, provided the training data is known and its adaptive nature will allow it to reduce the effect of strong
5 interferers from neighbouring cells and local narrow band interference.

Conclusions

We have shown that an FIR filter can be used to separate a desired signal from MAI in a DS-CDMA system with only a
10 small degradation in Gaussian noise performance. These filters can be approximated by trained adaptive filters. These sub-optimal filters may make practical receivers for a DS-CDMA cellular system.

THE DECORRELATING RECEIVER

15   The decorrelating detector is similar to the MMSE detector except that the noise term is not taken into account ie. the second term in equation {4} above is neglected in the formulae for $\emptyset_{yy}$.

RADIAL BASIS FUNCTION RECEIVERS

20   *Introduction:* In many DS-CDMA communications systems there are three sources of distortion when the signal arrives at the receiver, structured multiple access interference (MAI) from other users in the system, Gaussian noise which can often be extended to include unstructured interference and time
25 dispersion due to multipath propagation. The simplest DS-CDMA receiver structures are based on matched filters for a non-dispersive channel and RAKE receivers for multipath channels. The MAI performance of matched filter/RAKE receivers can be enhanced by applying cancellation at the expense of
30 increased receiver complexity [12]. Many proposed receiver structures are based on linear equaliser structures. Examples include decorrelating receivers which are based on the zero-forcing equaliser and those based on the MMSE equaliser [21] . We shall compare the above receiver structures with the
35 RBF receiver. This nonlinear receiver minimises the

probability of error when deciding on a data bit. It has
already been shown that an RBF equaliser has superior
performance to a linear equaliser in multipath channels for
conventional minimum bandwidth signalling [22] and that an RBF
5 filter has superior performance to a MMSE filter in a
non-dispersive CDMA system [23] . We shall show that the RBF
based receivers have the ability to considerably increase the
capacity of a DS-CDMA system when compared with other receiver
structures.

10   *AWGN channel system model:* To enable us to compare the
wide variety of receivers discussed above we shall first
consider an additive white Gaussian noise channel. Our system
will consist of U independent users each transmitting a
DS-CDMA signal which is chip and bit synchronous and with all
15 users transmitting equal power normalised to 1. The data bit
transmitted by user u during bit time k will be denoted by
$D_u(k)$ and the spreading code for user u by $C_{u,n}$. We will use n
to denote the chip number within the code which is an integer
between 0 and $(N-1)$ where N is the spreading sequence length
20 (processing gain) . The spreading codes used throughout are 7
chips long and randomly generated. Without loss of generality
we can assume that the desired user is user 0. The channel
model will be a simple additive white Gaussian noise (AWGN)
model with the noise time series denoted by $G(kN + i)$. Thus
25 the chip rate signal arriving at the receiver, denoted $Y(kN
+ i)$, will be:-

$$Y(kN + i) = \sum_{u=0}^{U-1} D_u(k)C_{u,i} + G(kN + i)$$

at the point where data bit k chip i is received. In the AWGN
case there is no need to consider i outside the range $0 \le i < N$
as outside this time the signal will contain no useful
30 information relating to data bit k.

AWGN channel receiver structures: We shall consider three
main receiver structures, matched filtering, the MMSE receiver
and the RBF receiver. In all cases we shall assume that the
codes and the signal powers are known at the receiver. In the

non-dispersive case the matched filter receiver is given by an N tap FIR filter whose co-efficients H_i are the spreading code for the desired receiver i.e.

$$H_i^{MF} = C_{0,i}$$

The MMSE receiver is also an N tap FIR filter, but the
5 co-efficients of this filter are given (in vector form) by [24] :-

$$\underline{H}^{MMSE} = \Phi_{yy}^{-1}\underline{\Phi}_{yx}$$

Φ_{yy} is the autocorrelation matrix of the cyclostationary input signal with dimensions NxN. Φ_{yx} is the cross correlation vector. The RBF receiver is a nonlinear filter whose estimate
10 of the data output is given by:-

$$\hat{D}_0 = sgn \sum_{i=1}^{n_c} w_i \exp\left(-\frac{\left(|\underline{y}(k) - \underline{c}_i|^2\right)}{2\sigma^2}\right)$$

where $\underline{y}(k)$ denotes the input vector consisting of the N chip spaced input samples at data bit time k and $\underline{c}_i$ are the centres. The centres are the noise free input vectors for all possible input data bit combinations. There are therefore n_c
15 $= 2^v$ centres. $|.|$ denotes the length of the enclosed vector (Euclidean norm). σ is the standard deviation of the noise. w_i is the value of D_0 associated with centre c_i. In our simulations we shall assume that the centres and weights are known at the receiver. For all three receiver cases we shall
20 also consider supplementing the receiver with a single stage of parallel cancellation similar to [12].

AWGN channel simulation results: The receiver structures above were simulated using Monte-Carlo simulation. The graphs show the $\log_{10}$ of the BER averaged over all active users in
25 the system plotted against the number of active users. Fig. 13 shows results for $E_b/N_0 = 9$ dB. These figures show that the performance of matched filtering becomes poor as the MAI (the number of active users) increases. The number of active users which gives an acceptable BER for a matched filter receiver
30 improves considerably with the addition of cancellation, as

this filter is clearly interference limited. The MMSE receiver
performs better than both the matched filter and the matched
filter with cancellation. It too improves with the addition
of cancellation, although the improvement is not as great as
5 adding cancellation to a matched filter. The RBF receiver is
considerably better than both the matched filter and the MMSE
receiver, with or without cancellation. However, the RBF
receiver is not improved by the addition of cancellation. This
is because in this scenario the RBF is the maximum likelihood
10 receiver and therefore cancellation cannot improve its
performance.

      Multipath channel system model: We shall now consider a
stationary multipath channel model. The channel we shall
consider has impulse response:-

15          $M(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2}$

All signals are assumed to pass through the same channel, as
would be the case in the downlink of a mobile radio system.
The received signal at the time data bit k, chip i is received
by the direct path and therefore becomes:-

$$Y(kN+i) = 0.3482\left(\sum_{u=0}^{U-1} D_u(k)C_{u,i} + G(kN+i)\right) +$$

$$0.8704\left(\sum_{u=0}^{U-1} D_u(k)C_{u,i-1} + G(kN+i-1)\right) + 0.3482\left(\sum_{u=0}^{U-1} D_u(k)C_{u,i-2} + G(kN+i-2)\right)$$

20 Note that if $i-x$ is negative then the $D_u(k)C_{u,i-x}$ is replaced by
$D_u(k-1)C_{u,N+i-x}$ and if i or $i-x$ is greater than N (if we extend
the receiver filter length beyond N) then $D_u(k)C_{u,i-x}$ is
replaced by $D_u(k+1)C_{u,i-N-x}$. This is a direct illustration of ISI
caused by the multipath channel.

25   *Multipath channel receiver structures:* We shall base all
our new receiver structures on a new set of input signal
samples $y(kN+i)$ where the range of i is extended to $0 \leq i < N+2$
to capture all the signal energy that originated from data bit
k. This combined with the multipath channel will result in ISI
30 from both the preceding and following data bit. The equivalent
of the matched filter for a multipath channel is an FIR filter

with N+2 taps where the taps are given by:-

$$H^{RAKE} = \underline{c}_0 * M$$

The * represents convolution and $\underline{c}_0$ and M represent the
spreading code of user 0 and impulse response of the channel
respectively. The MMSE receiver coefficients are given by the
5   same equation as before, however, the elements of the
$((N+2) \times (N+2))$ autocorrelation matrix Φ and the $(N+2)$ cross
correlation vector must be calculated according to the
derivation of the MMSE receiver taking into account the
multipath. The RBF receiver equation is also essentially
10 unchanged, except now the input and centre vectors are of
length $N+2$ and the number of centres is now 2^{3U} as all
possible combinations of the previous, current and next data
bit must be considered. This number of centres increases
rapidly with the number of active users U and the
15 computational load will rapidly become impractical.

Multipath channel simulation results: Graphs for the BER
performance of the receiver structures described in the
previous section are shown in Fig. 14 for E_b/N_0 = 9 dB. These
graphs show similar trends to the nondispersive case. The RAKE
20 receiver rapidly breaks down as the MAI interference increases
because the interfering users are not taken account of at all
in the RAKE receiver and are therefore treated as unstructured
noise. The MMSE receiver does considerably better than the
RAKE receiver. The RBF receiver performs better than either
25 the RAKE receiver or the MMSE receiver. However, it does
involve a considerable increase in computational complexity.
The RBF results are truncated at 5 users because the
calculation of 6 users would require evaluating the Euclidean
distance from 2^{18} centres for every data bit sent.

30   *Discussion and Conclusions:* We have shown that an RBF
receiver has greatly improved performance over the more
conventional matched filter and MMSE based receiver
structures. This performance increase is apparent in both AWGN
and multipath channels. However the computational complexity

and the number of variable parameters in a non-stationary environment presently make the RBF filter receiver impractical for mobile radio applications.

NEAR OPTIMUM RECEIVERS

5   Near Optimum receivers use similar methods to the Radial Basis Function receiver, usually with simplifications to the Radial Basis Function by dropping the exponential term and simplifying the Euclidean difference expression. These receivers can be implemented at the chip level or the bit

10 level (after preprocessing) and often involve the Viterbi forward programming algorithm to search for the optimum centre (path in the Viterbi algorithm).

CANCELLATION BASED RECEIVERS

The optimal receiver for a direct sequence code division

15 multiple access (DS-CDMA) system is the maximum likelihood sequence estimator [25][14] which is effectively an RBF receiver with infinite memory. However, this receiver's complexity rises exponentially with the number of users and is therefore impractical. At the other end of the complexity

20 scale, a matched filter is commonly used as the receiver in a DS-CDMA system. A matched filter is the optimal receiver only in additive white Gaussian noise (AWGN) and has very poor performance when the level of multiple access interference (MAI) from other users in the system is high.

25   Between these two extremes a range of sub-optimal receiver structures have been proposed. The majority of these can be split into two types. Cancellation structures are typically based on matched filtering followed by subtraction of the matched filter output from a delayed version of the

30 input signal [12][26][27]. Equaliser structures use a linear or decision feedback FIR filter with the co-efficients optimised according to varying criterion such as minimum mean square error (MMSE) or zero-forcing [18][20][28]. A comprehensive review of this area and a reference list can be

35 found in [21].

In this section the performance of a Wiener filter receiver is compared with that of a simple parallel canceller. The combination of the two techniques is also examined.

System description

5   The system to be considered consists of U independent equal power users transmitting both data bit and chip synchronously as shown in Fig. 15. The data bit transmitted by user u at bit time k will be denoted by $D_u(k)$ and the spreading code for user u will be denoted by $C_{u,n}$. n denotes

10   the chip within the code which is an integer between 0 and $(N-1)$ and N is the spreading sequence length (processing gain). The spreading codes used as an example throughout this paper are 64 chips long and randomly generated. Orthogonal (Walsh) or semi-orthogonal (Gold) codes would give better

15   performance, but would require longer simulation times to give meaningful BER results. In many communications systems the channel will destroy the orthogonality property of orthogonal codes anyway. Throughout this paper the chip and data values will be normalised to ±1. Without loss of generality we can

20   assume that the desired user is user 0. The channel under consideration will be a simple AWGN channel with the noise denoted by $G(kN+n)$ having variance σ^2. Thus the chip rate signal arriving at the receiver, denoted $Y(kN+n)$, will be:-

$$Y(kN + n) = \sum_{u=0}^{U-1} D_u(k)C_{u,n} + G(kN + n)$$

Receiver structures and theoretical performance

25   The four receiver structures shown in Fig. 16 will be examined, Receiver structure a) is the simple matched filter case. The receiver consists of an N tap FIR filter whose co-efficients h_n are a scaled version of the desired user's code, ie:-

$$h_n = \frac{C_{0,n}}{N}$$

30 The matched filter treats the MAI as noise and therefore, if the signal power of each user is one and the codes are random, the central limit theorem allows us to approximate the BER for

this receiver as:-

$$BER_{mf} = \text{erfc}\left(\sqrt{\frac{N}{\sigma^2 + (U-1)}} \right) \tag{1}$$

where erfc() is the complimentary error function.

Receiver structure b) is a simple parallel canceller using matched filters to despread each interfering signal and
5 reconstruct a replica which is subtracted from an appropriately delayed input signal, cancelling much of the interference. The remaining signal is then despread using a matched filter to the desired user. The reconstruction of the signal assumes that the power of the signal is known exactly.
10 Therefore an interfering signal which is is received with the correct sign will be cancelled exactly and an interfering signal received incorrectly will have its amplitude doubled (power quadrupled). A lower bound on the BER for this structure is given by:-

$$BER = \text{erfc}\left(\sqrt{\frac{N}{\sigma^2 + BER_{mf}(4.0)(U-1)}} \right) \tag{2}$$

15 This equation is only a lower bound for two reasons. Firstly the noise samples for each matched filter in the cancellation stage are the same, whereas the above equation assumes they are independent samples. Secondly, after cancellation the combined interference plus noise distribution of the remaining
20 signal is a poor approximation to Gaussian because it consists of the Gaussian noise and a limited number of enlarged interferering signals which will be a poor approximation to a Gaussian distribution.

Receiver structure c) is the Wiener, Levinson or MMSE
25 producing filter [15][1] . It is also an N tap FIR filter, but the co-efficients of this filter are given (in vector form) by [24]:-

$$\underline{h} = \Phi_{yy}^{-1}\Phi_{yx}$$

where

$$\underline{h} = \left[h_0, h_1, h_2 \cdots h_{N-1} \right]^T$$

The T superscript denotes a matrix transpose. Φ_{yy} is the autocorrelation matrix of the input signal with dimensions NxN, given by:-

$$\Phi_{yy} = \underline{Q}\underline{Q}^T + \sigma^2 \underline{I}$$

The matrix Q has dimensions NxU and has the codes as its
5 columns, ie:-

$$Q = \begin{bmatrix} C_{0,0} & C_{1,0} & C_{2,0} & . & . & C_{U-1,0} \\ C_{0,1} & C_{1,1} & C_{2,1} & . & . & C_{U-1,1} \\ C_{0,2} & C_{1,2} & C_{2,2} & . & . & C_{U-1,2} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ C_{0,N-1} & C_{1,N-1} & C_{2,N-1} & . & . & C_{U-1,N-1} \end{bmatrix}$$

$\underline{I}$ is the NxN identity matrix. Φ_{yx} is the cross correlation vector:-

$$\Phi_{yx} = [C_{0,0}, C_{0,1}, C_{0,2} \cdots C_{0,N-1}]^T$$

The MMSE is given by:-

$$MMSE = 1 - \underline{h}^T \Phi_{yx}$$

and the BER performance (derivation A) is:-

$$BER_{Wiener} = erfc\left(\sqrt{\left(\frac{(\underline{h}^T\Phi_{yx})^2}{-(\underline{h}^T\Phi_{yx})^2 + (2.0)\underline{h}^T\Phi_{yx} - 1.0 + MMSE} \right)} \right) \tag{3}$$

10 To obtain average BER performance, this equation must be
averaged over all the users in the system to give BER_{Wiener}.

Receiver structure d) is a parallel canceller using
Wiener filters for the initial estimate of the interferers
data bits. By analogy with equation 2, a lower bound on the
15 BER performance of this structure is given by:-

$$BER = erfc\left(\sqrt{\frac{N}{\sigma^2 + \overline{BER}_{Wiener}(4.0)(U-1)}} \right) \tag{4}$$

Results

All simulated results in this section are for 60,000 data
bits, with the BER averaged over all users. Graphs show the
average BER for all users in the system plotted against number
20 of users. The sequence length is 64. The signal to Gaussian
noise ratio is exoressed as:-

$$E_b/N_0 = \frac{N}{2\sigma^2}$$

Fig. 17 shows a comparison between the theoretical
results derived in above and simulation results. The results
show that equations (1) and (3) give good and unbiased

estimates of the simulated results for the matched filter and
Wiener filter respectively. The deviation of the simulated
results for the matched filter from equation (1) with a low
number of users is caused by the interference distribution
5 only being approximately Gaussian and the limited number of
simulated points. Equations (2) and (4) however only give an
approximation to the actual performance of the two
cancellation receivers. This estimate is biased towards a
lower BER for reasons that were discussed above.

10   Fig. 18 shows simulated results for a range of signal to
noise ratios. These results show that a matched filter is only
a good receiver structure when Gaussian noise is the dominant
source of interference. This only applies when there are very
few interfering users and the signal to noise ratio is low.

15   A single stage of parallel cancellation based on matched
filters performs better than a matched filter except at
extremely high levels of interference, where the BER is too
low for most communications systems anyway. The improvement
is fairly significant, with E_b/N_0 = 9 dB, the cancellation
20 receiver can support three times as many users as the matched
filter if the required BER is 10^{-2}.

     The Wiener filter will always perform better than the
matched filter. The only case where the performance of these
two receiver systems is the same is when there is no MAI
25 interference and the Wiener filter becomes a scaled version
of the matched filter. The Wiener filter also performs
significantly better than the matched filter based
cancellation receiver. With E_b/N_0 = 9 dB and a required BER of
10^{-2}, the Wiener filter will allow approximately six times as
30 many users as a matched filter and twice as many as the
matched filter based cancellation receiver. The computational
complexity of the Wiener filter receiver is identical to the
matched filter receiver, and significantly less than the
matched filter based cancellation receiver. This assumes that
35 the Wiener co-efficients are calculated in advance.

The Wiener filter based cancellation receiver gives the best performance of all. It will allow approximately seven times the number of users if E_b/N_0 = 9 dB and the required BER is 10^{-2}, compared with the matched filter receiver.

5   Note that in all cases there is a significant variation in performance between users. For example, using the Wiener filter receiver with 48 users and E_b/N_0 = 9 dB, the average BER is 0.00940 but the user with the best spreading sequence experiences a BER of 0.001167 and the user with the worst
10 spreading sequence experiences a BER of 0.038092. This variation is due to the cross correlation properties of the code set and will also apply to an orthogonal code set which has lost its orthogonality due to a multipath channel.

Discussion and Conclusions
15   In this paper we have shown that cancellation and Wiener filtering both perform much better in MAI than a simple DMF. Wiener filters in particular offer a large increase in the number of users that can be accommodated. The two ideas can be successfully combined and provide a better performance than
20 either on its own.

There is a cost to pay for improved performance. The cancellation receiver introduces a delay of one data bit and requires a substantial increase in either computation or hardware. Both the Wiener filter and the cancellation receiver
25 require knowledge of the number and spreading sequences of the interfering users at the receiver, and the Wiener filter also requires a priori knowledge of the signal to noise ratio. If the signal to noise ratio is very high, calculation of the inverse of the autocorrelation matrix for the Wiener filter
30 can be difficult as it becomes close to singular. In a computer network application the calculation of the Wiener filter in advance may be feasible, but in a mobile communications application the signal to noise ratio and the number of users varies rapidly with time. However, a good
35 approximation to the Wiener filter can often be obtained using

an adaptive algorithm.

Derivation A

To calculate the BER for the Wiener filter we require to find a relationship between the MMSE and the variance at the output of the filter.

$$MMSE = E[(D_0(k) - \hat{x})^2] = E[D_0^2(k)] - (2.0)E[D_0(k)\hat{x}] + E[\hat{x}^2]$$

where $E[]$ denotes the expected value and x denotes the filter output. $D_o(k)$ takes the values ± 1, therefore:-

$$MMSE = 1 - (2.0)\underline{h}^T \Phi_{yx} + E[\hat{x}^2] \qquad [A.1]$$

The variance of the filter output is given by:-

$$var(\hat{x}) = E[(\bar{x} - \hat{x})^2] = E[\bar{x}^2] - (2.0)E[\bar{x}\hat{x}] + E[\hat{x}^2]$$

where $\bar{x}$ denotes $E[x]$.

$$var(\hat{x}) = (\underline{h}^T \Phi_{yx})^2 - (2.0)(\underline{h}^T \Phi_{yx})^2 + E[\hat{x}^2] \qquad [A.2]$$

Subtracting [A.1] from [A.2] and rearranging gives:-

$$var(\hat{x}) = -(\underline{h}^T \Phi_{yx})^2 + (2.0)\underline{h}^T \Phi_{yx} - 1.0 + MMSE$$

Therefore, assuming the output of the Wiener filter has a Gaussian distribution, the BER is given by:-

$$BER_{Wiener} = erfc\left(\sqrt{\frac{(\underline{h}^T \Phi_{yx})^2}{var(\hat{x})}}\right) = erfc\left(\sqrt{\frac{(\underline{h}^T \Phi_{yx})^2}{-(\underline{h}^T \Phi_{yx})^2 + (2.0)\underline{h}^T \Phi_{yx} - 1.0 + MMSE}}\right)$$

VOLTERRA RECEIVER

The Volterra receiver is made up by a power series expansion of the received signal followed by a linear filter such as MMSE where the Volterra expanded signal v is used instead of the original received signal y. For a DS-CDMA system with received input signal $y(kN+n)$, the output of the third order Volterra expansion is given by:-

$$\hat{D} = sgn \sum_{a=0}^{N-1} h_1(a)y(kN + n - a) + \sum_{a=0}^{N-1} \sum_{b=0}^{N-1} h_2(a,b)y(kN + n - a)y(kN + n - b)$$

$$+ \sum_{a=0}^{N-1} \sum_{b=0}^{N-1} \sum_{c=0}^{N-1} h_3(a,b,c)y(kN + n - a)y(kN + n - b)y(kN + n - c)$$

where the h co-efficients are estimated using a linear technique such as MMSE with the autocorrelation and crosscorrelation matrices replaced with the autocorrelation and crosscorrelation matrices of the power series expanded matrices.

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CLAIMS

1. A direct sequence code division multiple access receiver comprising an adaptive filter controlled by an adaptive algorithm for filtering data which has been
5 multiplied by a spreading code and filtered by a channel filter, the adaptive filter having a length appropriate to model the inverse of the channel filter, and a multiuser detector operating on the output of the adaptive filter.

2. A receiver according to claim 1, wherein the
10 algorithm is trained using the signal of a desired user.

3. A receiver according to claim 1 or 2, wherein the algorithm is trained using a composite signal from more than one user.

4. A receiver according to claim 1, 2 or 3, wherein
15 the multiuser detector is of the minimum mean squared error type.

5. A receiver according to claim 1, 2 or 3, wherein the multiuser detector is of the zero forcing (decorrelating) type.

20   6. A receiver according to claim 1, 2 or 3, wherein the multiuser detector is of the Volterra type.

7. A receiver according to claim 1, 2 or 3, wherein the multiuser detector is of the Radial Basis Function type.

8. A receiver according to claim 1, 2 or 3, wherein
25 the multiuser detector is of the cancellation type.

9. A receiver according to claim 1, 2 or 3, wherein the multiuser detector is of the near optimum decoding type.

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10. A receiver according to any preceding claim, wherein the algorithm can comprises the least mean squares algorithm.

11. A receiver according to any one of claims 1 to 9, 5 wherein the algorithm comprises the recursive least squares algorithm.

12. A receiver according to any one of claims 1 to 9, wherein the algorithm comprises the fast a-posterion error sequential technique algorithm.

10 13. A receiver according to any one of claims 1 to 9, wherein the algorithm comprises the stabilised fast a-posterion error sequential technique algorithm.

14. A receiver according to claim 12 or 13, wherein said algorithm is used in combination with the Fast Newton 15 algorithm.

FIG. 1

(PRIOR ART)

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. 11

FIG. 12

- ◆ MATCHED FILTER
- + MATCHED FILTER WITH CANCELLATION
- □ MMSE RECEIVER
- ✕ MMSE RECEIVER WITH CANCELLATION
- △ RBF RECEIVER
- ✱ RBF RECEIVER WITH CANCELLATION

FIG. 13

- ◆ RAKE RECEIVER
- + MMSE RECEIVER
- □ RBF RECEIVER

FIG. 14

DESIRED USER

DATA D_0 → ⊗

SPREADING CODE
$C_{0,0} - C_{0,N}$

ADDITIVE WHITE GAUSSIAN NOISE

$G(n)$

INTERFERING USER

DATA D_1 → ⊗

SPREADING CODE
$C_{1,0} - C_{1,N}$

DATA D_2 → ⊗

SPREADING CODE
$C_{2,0} - C_{2,N}$

DATA D_0 → ⊗

SPREADING CODE
$C_{U-1,0} - C_{u-1,N}$

Σ → $T(n)$ → Σ → $Y(n)$

FIG. 15

a) SIMULATED MATCHED FILTER ◇
EQUATION (1) − − −
b) SIMULATED MATCHED FILTER BASED CANCELLATION +
EQUATION (2) ·······
c) SIMULATED WIENER FILTER □
EQUATION (3) − · − ·
d) SIMULATED WIENER FILTER BASED CANCELLATION ✕
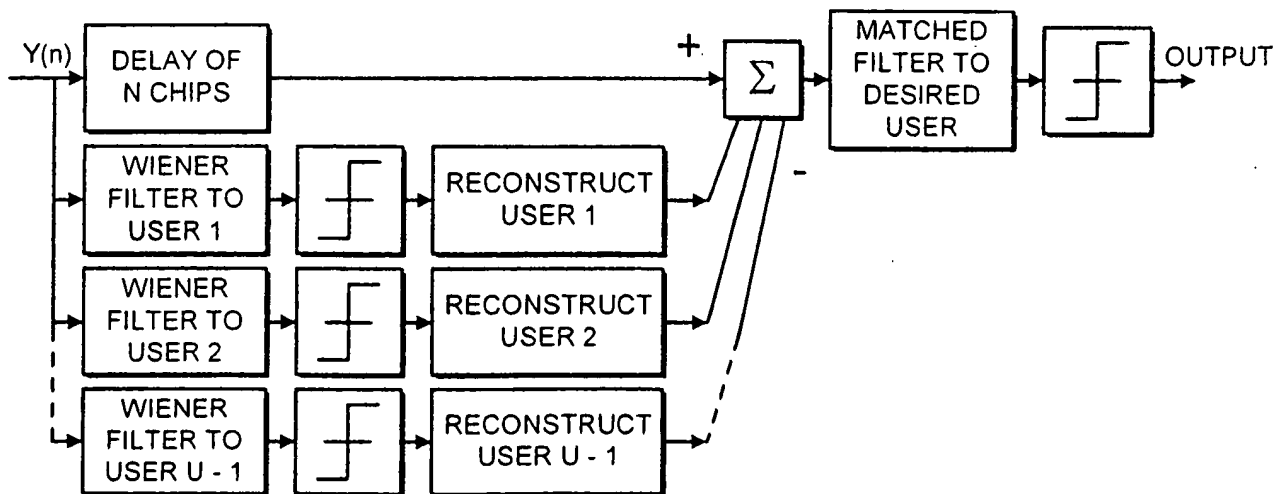EQUATION (4) ———

LOG BER

NUMBER OF USERS

FIG. 17

10 / 12

a) MATCHED FILTER
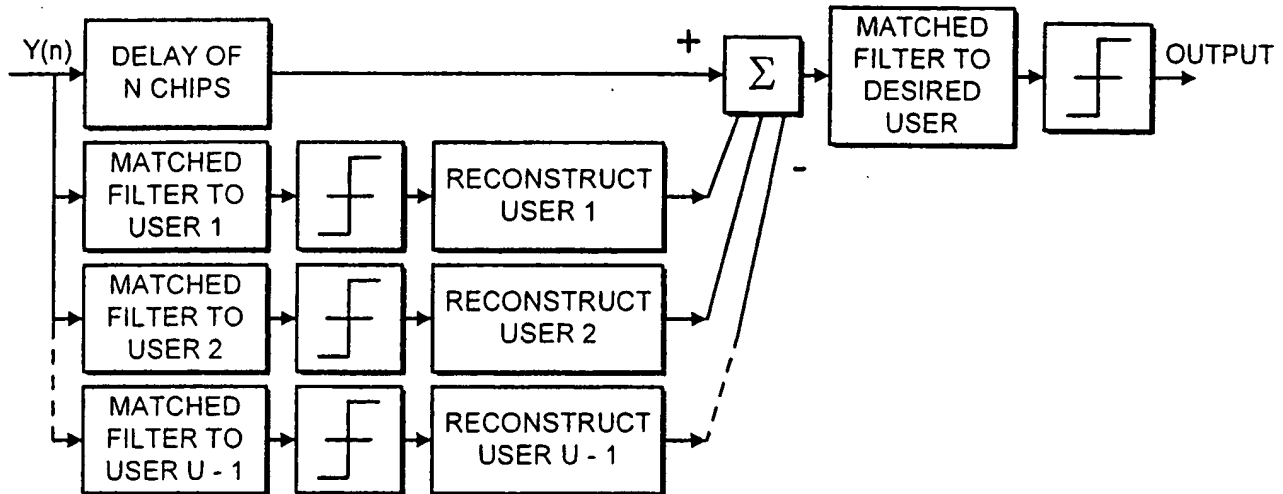
b) MATCHED FILTER BASED CANCELLATION
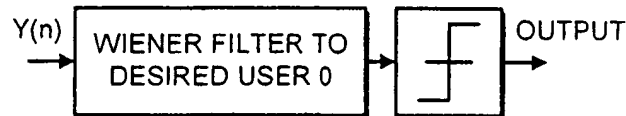
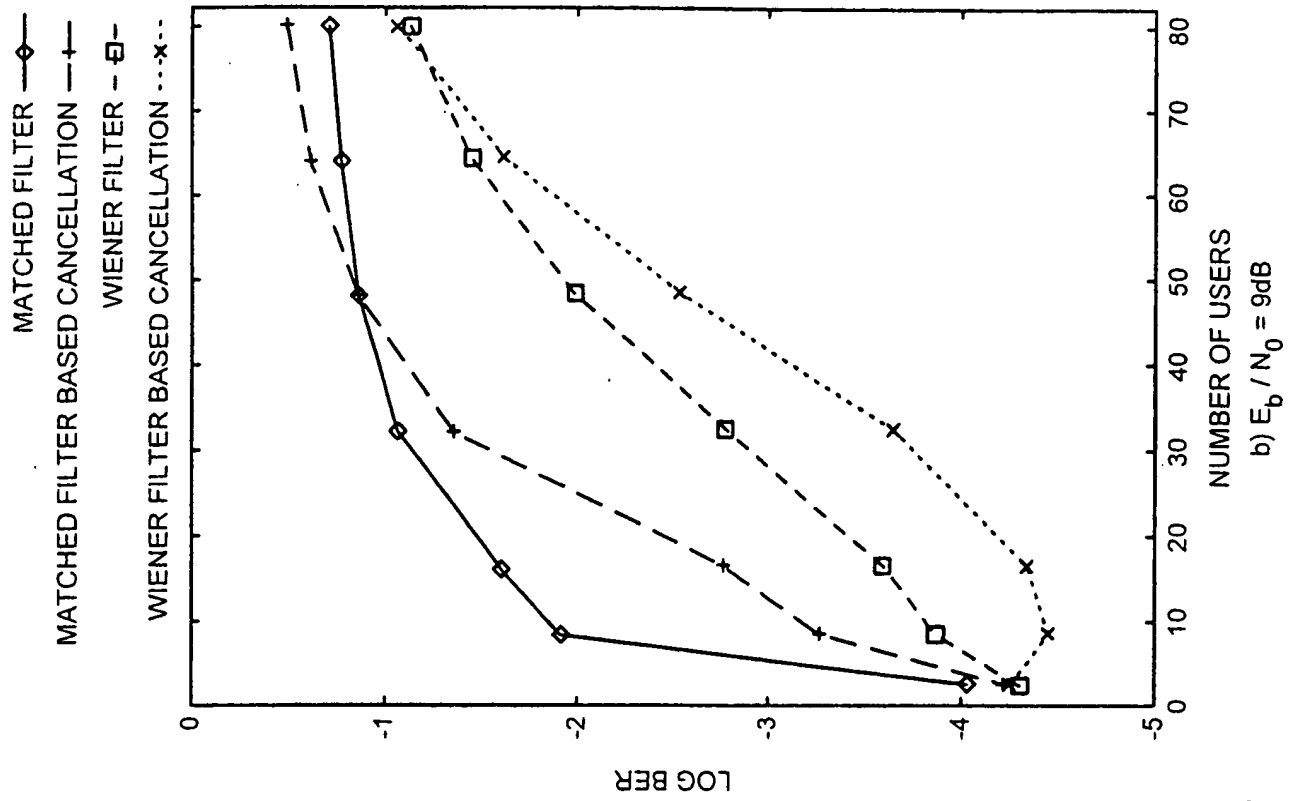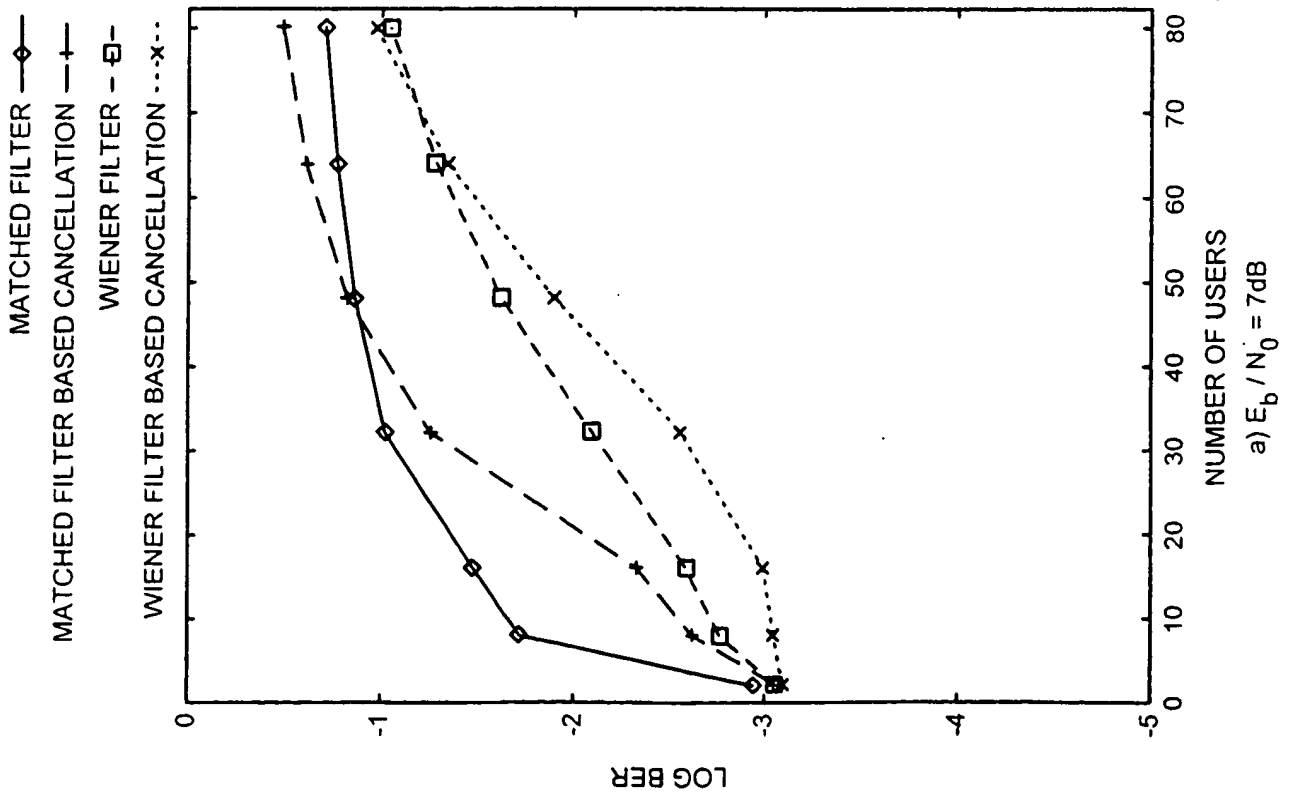c) WIENER FILTER

d) WIENER FILTER BASED CANCELLATION
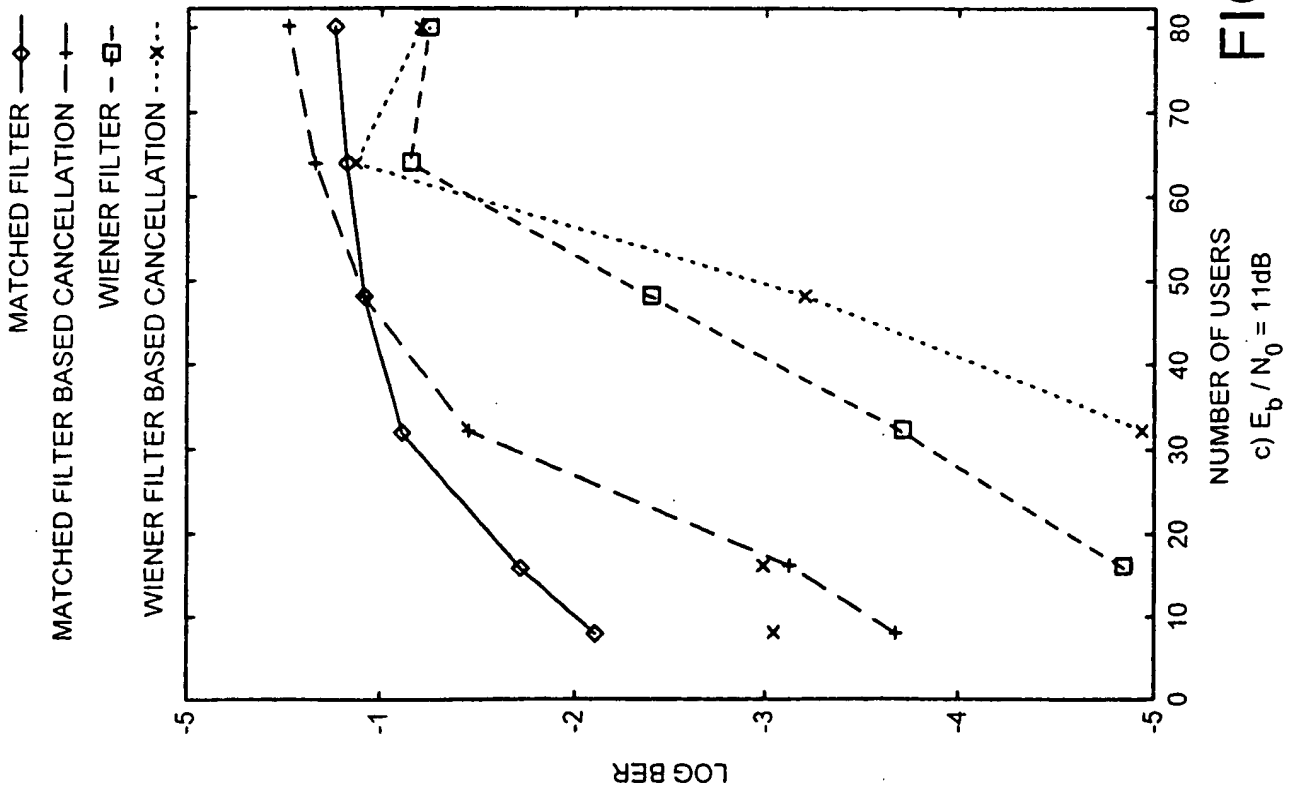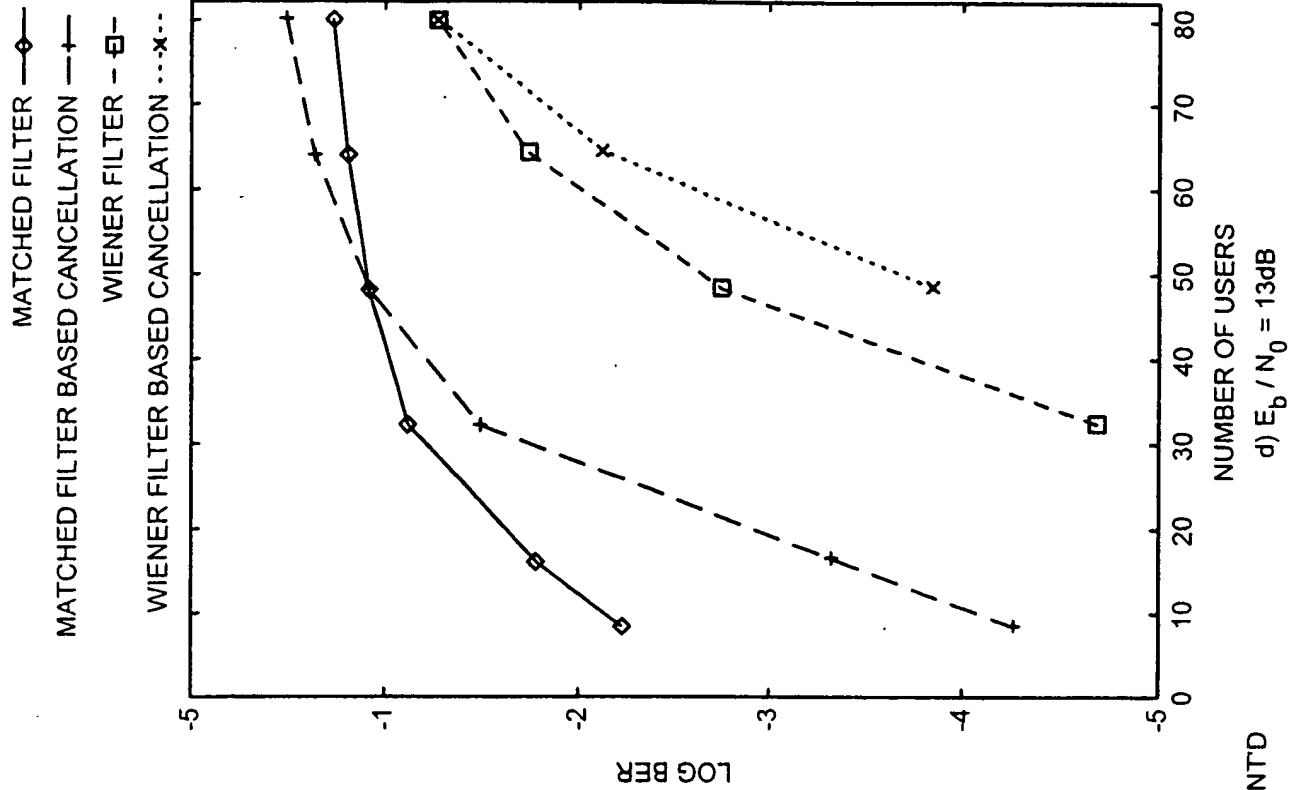
FIG. 16

FIG. 18

FIG. 18 CONTD

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7   H04B1/707   H04L25/03

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7   H04B   H04L

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	US 5 692 006 A (ROSS ARTHUR) 25 November 1997 (1997-11-25) column 3, line 28 – line 55 column 4, line 8 –column 6, line 8; figure 1 ---	1,10
A	WO 94 29985 A (MOTOROLA INC) 22 December 1994 (1994-12-22) page 8, line 26 –page 10, line 29; figure 5 -----	1,10

☐ Further documents are listed in the continuation of box C.

☒ Patent family members are listed in annex.

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"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

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Date of the actual completion of the international search	Date of mailing of the international search report
23 June 2000	30/06/2000

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Inte onal Application No

PCT/GB 00/00649

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